Ratio and Proportion

Practice set 4.1

Q. 1. From the following pairs of numbers, find the reduced form of ratio of first number to second number.

i. 72, 60
ii. 38, 57
iii. 52, 78

Answer : (i) 72,60

Reduced form of ratio of first number to second number:

$$\frac{72}{60} = \frac{12 \times 6}{12 \times 5}$$

(Break the expression in order to simplify it further)

$$= \frac{6}{5}$$

(ii) 38,57

Reduced form of ratio of first number to second number:

$$\frac{38}{57} = \frac{19 \times 2}{19 \times 3}$$

(Break the expression in order to simplify it further)

$$= \frac{2}{3}$$

(iii) 52, 78

Reduced form of ratio of first number to second number:

$$\frac{52}{78} = \frac{26 \times 2}{26 \times 3}$$

(Break the expression in order to simplify it further)

$$= \frac{2}{3}$$

Q. 2. Find the reduced form of the ratio of the first quantity to second quantity.

i. 700 Rs, 308Rs.
ii. 14Rs, 12 Rs.40 paise.
iii. 5 litre, 2500 ml
iv. 3 years 4 months, 5 years 8 months
v. 3.8 kg, 1900 gm
vi. 7 minutes 20 seconds, 5minutes 6 seconds.

Answer : (i) Reduced form of the ratio of 700Rs and 308Rs is:

$$\frac{700\ \text{Rs}}{308\ \text{Rs}} = \frac{(28 \times 25)\text{Rs}}{(28 \times 11)\text{Rs}}$$

(Break the expression in order to simplify it further)

$$= \frac{25\text{Rs}}{11\text{Rs}}$$

(ii) Reduced form of the ratio of 14Rs and 12.40Rs is:

$$\frac{14\ \text{Rs}}{12.40\ \text{Rs}}$$

Multiply denominator and numerator by 100:

$$= \frac{1400\ \text{Rs}}{1240\ \text{Rs}}$$

Divide numerator and denominator by 10:

$$= \frac{140\text{Rs}}{124\text{Rs}}$$

$$= \frac{(35 \times 4)\text{Rs}}{(31 \times 4)\text{Rs}}$$

(Break the expression in order to simplify it further)

$= \frac{35Rs}{31Rs}$

(iii) 5 litre = 5000 ml

∴ Reduced form of the ratio of 5000 ml and 2500 ml is:

$\frac{5000}{2500} = \frac{2500 \times 2}{2500}$

(Break the expression in order to simplify it further)

$= \frac{2}{1}$

(iv) 3 years = 3 × 12 = 36 months

∴ 3 years 4 months = 40 months

5 years = 5 × 12 = 60

∴ 5 years 8 months = 68 months

∴ Reduced form of the ratio of 40 months and 68 months is:

$\frac{40}{68} = \frac{4 \times 10}{4 \times 17}$

(Break the expression in order to simplify it further)

$= \frac{10}{17}$

(v) 3.8 kg = 3.8 × 1000 = 3800 gm

∴ Reduced form of the ratio of 3800 gm and 1900 gm is:

$\frac{3800}{1900} = \frac{1900 \times 2}{1900 \times 1}$

(Break the expression in order to simplify it further)

$= \frac{2}{1}$

(vi) 7 minutes = 7 × 60 = 420 seconds

∴ 7 minutes 20 seconds = 440 seconds

5minutes = 5 × 60 = 300 seconds

∴ 5 minutes 6 seconds = 306 seconds

∴ Reduced form of the ratio of 440 seconds and 306 seconds is:

$$\frac{440}{306} = \frac{220 \times 2}{153 \times 2}$$

(Break the expression in order to simplify it further)

$$= \frac{220}{153}$$

Q. 3 .Express the following percentages as ratios in the reduced form.

(i) 75 : 100
(ii) 44 : 100
(iii) 6.25%
(iv) 52 : 100
(v) 0.64%

Answer : (i) Reduced form of the ratio of 75:100 is:

$$\frac{75}{100} = \frac{25 \times 3}{25 \times 4} = \frac{3}{4}$$

(ii) Reduced form of the ratio of 44:100 is:

$$\frac{44}{100} = \frac{4 \times 11}{4 \times 25} = \frac{11}{25}$$

(iii) Reduced form of 6.25% is:

$$6.25\% = \frac{6.25}{100} = \frac{625}{10000} = \frac{625 \times 1}{625 \times 16} = \frac{1}{16}$$

(iv) Reduced form of the ratio of 52:100 is:

$$\frac{52}{100} = \frac{13 \times 4}{25 \times 4} = \frac{13}{25}$$

(v) Reduced form of 0.64% is:

$$0.64\% = \frac{.64}{100}$$

$$= \frac{64}{10000}$$

$$= \frac{16 \times 4}{625 \times 16}$$

$$= \frac{4}{625}$$

Q. 4. Three persons can build a small house in 8 days. To build the same house in 6 days, how many persons are required?

Answer :

No. of persons required to build a house in 8 days =3
No. of persons required to build a house in 1 day =3×8 = 24
No. of persons required to build a house in 6 days = 24/6 = 4

∴ 4 persons are required to build the same house.

Q. 5. Convert the following ratios into percentage.

(i) 15 : 25
(ii) 47 : 50
(iii) 7/10
(iv) 546/600
(v) 7/16

Answer : (i) 15 : 25

= 15/25

= ((15/25) × 100)%

= (15 × 4) %

= 60 %

(ii) 47 : 50

= 47/50

= ((47/50) × 100)%

= (47 × 2) %

= 94 %

(iii) 7/10

= ((7/10) × 100)%

= (7 × 10) %

= 70 %

(iv) 546/600

= ((546/600) × 100)%

= (546/6) %

= 91 %

(v) 7/16

= ((7/16) × 100)%

= (7 × 6.25) %

= 43.75 %

Q. 6. The ratio of ages of Abha and her mother is 2 : 5. At the time of Abha's birth her mother's age was 27 year. Find the present ages of Abha and her mother.

Answer : Let age of Abha = x years

∴ Age of Abha's mother at her birth = (x + 27) years

Ratio of Abha's and her mother's age = 2:5

$$\frac{\text{Abha's age}}{\text{Abha's mother's age}} = \frac{2}{5}$$

$\Rightarrow \frac{x}{x+27} = \frac{2}{5}$

Cross multiply and get:

5x = 2(x + 27)

⇒ 5x = 2x + 54

⇒ 5x – 2x = 54

⇒ 3x = 54

⇒ x = 54/3

⇒ x = 18

∴ Age of Abha = 18 years

Age of Abha’s mother = 18 + 27 = 45 years

Q. 7. Present ages of Vatsala and Sara are 14 years and 10 years respectively. After how many years the ratio of their ages will become 5:4?

Answer : Given: Present age of Vatsala = 14 years

Present age of Sara = 10 years

Let after x years, the ratio of their ages will be 5:4.

∴ Age of Vatsala after x years = (14 + x) years

Age of Sara after x years = (10 + x) years

Ratio of their ages = 5:4

$\therefore \frac{14+x}{10+x} = \frac{5}{4}$

On cross multiplying, we get:

56 + 4x = 50 + 5x

⇒ 5x – 4x = 56 – 50

$\Rightarrow$ x = 6

$\therefore$ After 6 years, their ages will be 20 years and 16 years and ratio of their ages will be 5:4.

Q. 8. The ratio of present ages of Rehana and her mother is 2 : 7. After 2 years, the ratio of their ages will be 1 : 3. What is Rehana's present age?

Answer : Given: Ratio of present age of Rehana and her mother = 2:7

So, let present age of Rehana = 2x

$\therefore$ Present age of her mother = 7x

After 2 years, age of Rehana = (2x + 2) years

After 2 years, age of her mother = (7x + 2) years

Now, given that after two years ratio of their ages will be 1:3.

$$\therefore \frac{2x+2}{7x+2} = \frac{1}{3}$$

On cross multiplying, we get:

6x + 6 = 7x + 2

$\Rightarrow$ 7x – 6x = 6 – 2

$\Rightarrow$ x = 4

$\therefore$ Present age of Rehana = 2x = (2 × 4) years = 8 years

Practice set 4.2

Q. 1. Using the property $\frac{a}{b} = \frac{ak}{bk}$, fill in the blanks substituting proper numbers in the following.

(i) $\frac{5}{7} = \frac{...}{28} = \frac{35}{...} = \frac{...}{3.5}$

(ii) $\frac{9}{14} = \frac{4.5}{...} = \frac{...}{42} = \frac{...}{3.5}$

Answer :

(i) Let $\frac{5}{7} = \frac{x}{28} = \frac{35}{y} = \frac{z}{3.5}$

∴ on comparing first two equalities, we get:

5/7 = x/28

Cross multiply and get:

7x = 28 × 5

⇒ x = 4 × 5 = 20

Now, compare the first and third equalities and get:

5/7 = 35/y

Cross multiply and get:

5y = 7 × 35

⇒ y = 7 × 7 = 49

Now, compare the first and fourth equalities and get:

5/7 = z/3.5

Cross multiply and get:

7z = 5 × 3.5

⇒ 7z = 5 × (35/10)

⇒ z = 5 × (5/10)

⇒ z = 25/10 = 2.5

$\therefore \frac{5}{7} = \frac{20}{28} = \frac{35}{49} = \frac{2.5}{3.5}$

(ii) Let $\frac{9}{14} = \frac{4.5}{x} = \frac{y}{42} = \frac{z}{3.5}$

∴ On comparing first two equalities, we get:

9/14 = 4.5/x

Cross multiply and get:

9x = 14 × 4.5

⇒ x = 14 × 0.5 = 7

Now, compare the first and third equalities and get:

9/14 = y/42

Cross multiply and get:

14y = 9 × 42

⇒ y = 9 × 3 = 27

Now, compare the first and fourth equalities and get:

9/14 = z/3.5

Cross multiply and get:

14z = 9 × 3.5

⇒ z = 9 × (3.5/14)

⇒ z = 9 × (0.25)

⇒ z = 2.25

$\therefore \frac{9}{14} = \frac{4.5}{7} = \frac{27}{42} = \frac{2.25}{3.5}$

Q. 2. Find the following ratios.

(i) The ratio of radius to circumference of the circle.

(ii) The ratio of circumference of circle with radius r to its area.
(iii) The ratio of diagonal of a square to its side, if the length of side is 7 cm.
(iv) The lengths of sides of a rectangle are 5 cm and 3.5 cm. Find the ratio of its perimeter to area.

Answer : (i) Let r be the radius of a circle.

Circumference of circle = 2πr

Ratio of radius to circumference of circle = r/2πr

= 1/2π

= 1 : 2π

(ii) Let r be the radius of a circle.

Circumference of circle = 2πr

Area of the circle =$2\pi r^2$

Ratio of radius to circumference of circle = $2\pi r/\pi r^2$

= 2/r

= 2 : r

(iii) Side of square = 7 cm

Diagonal of square = √2 × side = 7√2 cm

Ratio of diagonal of a square to its side = 7/7√2

= 1/√2

= 1 : √2

(iv) Length of rectangle = 5 cm

Breadth of rectangle = 3.5 cm

Perimeter of rectangle = 2(Length + Breadth)

= 2(5+3.5)

= 2(8.5)

= 17 cm

Area of rectangle = Length × Breadth

= 5 × 3.5

= 16.5 cm^2

Ratio of Perimeter to area of rectangle

= 17/16.5

= 170/165

= 34/33

Q. 3. Compare the following pairs of ratios.

i. $\frac{\sqrt{5}}{3}, \frac{3}{\sqrt{7}}$ **ii.** $\frac{3\sqrt{5}}{5\sqrt{7}}, \frac{\sqrt{63}}{\sqrt{125}}$

iii. $\frac{5}{18}, \frac{17}{121}$ **iv.** $\frac{\sqrt{80}}{\sqrt{48}}, \frac{\sqrt{45}}{\sqrt{27}}$

v. $\frac{9.2}{5.1}, \frac{3.4}{7.1}$

Answer : (i) Given ratios are

$\frac{\sqrt{5}}{3}, \frac{3}{\sqrt{7}}$

Step I: Make the second term of both the ratios equal.

Multiply and divide first ratio by √7:

$$\frac{\sqrt{5} \times \sqrt{7}}{3 \times \sqrt{7}} = \frac{\sqrt{35}}{3\sqrt{7}}$$

Multiply and divide second ratio by 3:

$$\frac{3 \times 3}{\sqrt{7} \times 3} = \frac{9}{3\sqrt{7}}$$

Step II: Compare the first terms (numerators) of the new ratios.

Since the denominators of new ratios are equal, compare the numerators of the new ratios:

Since, 9>√35, therefore $\frac{9}{3\sqrt{7}} > \frac{\sqrt{35}}{3\sqrt{7}}$.

Therefore the second ratio is greater than the first ratio according to the ratio comparison rules.

$\Rightarrow \frac{\sqrt{5}}{3} < \frac{3}{\sqrt{7}}$

(ii) Given ratios are

$\frac{3\sqrt{5}}{5\sqrt{7}}, \frac{\sqrt{63}}{\sqrt{125}}$

Step I: Make the second term of both the ratios equal.

Multiply and divide first ratio by √5:

$$\frac{3\sqrt{5} \times \sqrt{5}}{5\sqrt{7} \times \sqrt{5}} = \frac{15}{5\sqrt{35}}$$

Multiply and divide second ratio by √7:

$$\frac{\sqrt{63} \times \sqrt{7}}{\sqrt{125} \times \sqrt{7}} = \frac{3 \times \sqrt{7} \times \sqrt{7}}{5 \times \sqrt{5} \times \sqrt{7}} = \frac{21}{5\sqrt{35}}$$

Step II: Compare the first terms (numerators) of the new ratios.

Since the denominators of new ratios are equal, compare the numerators of the new ratios:

Since, 21>15, therefore $\frac{21}{5\sqrt{35}} > \frac{15}{5\sqrt{35}}$.

Therefore the second ratio is greater than the first ratio according to the ratio comparison rules.

$\Rightarrow \frac{3\sqrt{5}}{5\sqrt{7}} < \frac{\sqrt{63}}{\sqrt{125}}$

(iii) Given ratios are

$\frac{5}{18}, \frac{17}{121}$

Step I: Make the second term of both the ratios equal.

Multiply and divide first ratio by 121:

$$\frac{5 \times 121}{18 \times 121} = \frac{605}{18 \times 121}$$

Multiply and divide second ratio by 18:

$$\frac{17 \times 18}{121 \times 18} = \frac{306}{18 \times 121}$$

Step II: Compare the first terms (numerators) of the new ratios.

Since the denominators of new ratios are equal, compare the numerators of the new ratios:

Since, $605 < 306$, therefore $\frac{605}{18 \times 121} > \frac{306}{18 \times 121}$.

Therefore, the first ratio is greater than the second ratio according to the ratio comparison rules.

$\Rightarrow \frac{5}{18} > \frac{17}{121}$

(iv) Given ratios are

$\frac{\sqrt{80}}{\sqrt{48}}, \frac{\sqrt{45}}{\sqrt{27}}$

Simplifying the ratios, we get:

$$\frac{\sqrt{16 \times 5}}{\sqrt{16 \times 3}}, \frac{\sqrt{9 \times 5}}{\sqrt{9 \times 3}} = \frac{\sqrt{5}}{\sqrt{3}}, \frac{\sqrt{5}}{\sqrt{3}}$$

Since, the denominators of both the terms are same; compare the first terms (numerators) of the new ratios.

Since the denominators of new ratios are equal, compare the numerators of the new ratios:

Since, $\sqrt{5} = \sqrt{5}$, therefore $\frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{3}}$.

Therefore, both the ratios are equal, according to the ratio comparison rules.

$\Rightarrow \frac{\sqrt{80}}{\sqrt{48}} = \frac{\sqrt{45}}{\sqrt{27}}$

(v) Given ratios are

$\frac{9.2}{5.1}, \frac{3.4}{7.1}$

Simplifying the ratios, we get:

$\frac{92}{51}, \frac{34}{71}$ (Multiply the numerator and denominator of both the ratios by 10)

Step I: Make the second term of both the ratios equal.

Multiply and divide first ratio by 71:

$$\frac{92 \times 71}{51 \times 71} = \frac{6532}{51 \times 71}$$

Multiply and divide second ratio by 51:

$$\frac{34 \times 51}{71 \times 51} = \frac{1734}{51 \times 71}$$

Step II: Compare the first terms (numerators) of the new ratios.

Since the denominators of new ratios are equal, compare the numerators of the new ratios:

Since, $6532 > 1734$, therefore $\frac{6532}{51 \times 71} > \frac{1734}{51 \times 71}$.

Therefore the first ratio is greater than the second ratio according to the ratio comparison rules.

$\Rightarrow \frac{9.2}{5.1} > \frac{3.4}{7.1}$

Q. 4 A. ABCD is a parallelogram. The ratio of ∠A and ∠B of this parallelogram is 5 : 4. Find the measure of ∠B.

Answer : Given: ∠A : ∠B = 5:4

Let the measure of ∠A = 5x

Then ∠B = 4x

Since the adjacent angles of a parallelogram are complementary, therefore ∠A +∠B = 180°

⇒ 5x + 4x = 180°

⇒ 9x = 180°

⇒ x = 20°

∴ ∠ A = 5x = 5 × 20° = 100°

∴ ∠ B = 4x = 4 × 20° = 80°

Q. 4 B. The ratio of present ages of Albert and Salim is 5 : 9. Five years hence ratio of their ages will be 3 : 5. Find their present ages.

Answer : Given: Ratio of present age of Albert and Salim=5:9

So, let present age of Albert = 5x years

∴ Present age of Salim = 9x years

Five years later, age of Albert = (5x + 5) years

Five years later, age of Salim = (9x + 5) years

Given that this ratio of their ages is 3:5.

∴ According to the given problem:

$$\frac{5x+5}{9x+5}=\frac{3}{5}$$

⇒ 25x + 25 = 27x + 15

⇒ 27x – 25x = 25 – 15

⇒ 2x = 10

⇒ x = 5

∴ Present age of Albert = 5x = 5 × 5 = 25 years

Present age of Salim = 9x = 9 × 5 = 45 years

Q. 4 C. The ratio of length and breadth of a rectangle is 3 : 1, and its perimeter is 36 cm. Find the length and breadth of the rectangle.

Answer : Let l and b be the length and breadth of rectangle.

Given that Length : Breadth = 3:1

Perimeter of rectangle = 36

Let length of rectangle = l = 3x cm

∴ Breadth of the rectangle = b = x cm

Perimeter of rectangle = 2(l + b) = 36 cm

∴ 2(3x + x) = 36

⇒ 2(4x) = 36

⇒ 8x = 36

⇒ x = 36/8 = 4.5

∴ Breadth of rectangle = x = 4.5 cm

Length of rectangle = 3x = 3 × 4.5 = 13.5 cm

Q. 4 D. The ratio of two numbers is 31 : 23 and their sum is 216. Find these numbers.

Answer : Let a and b be two numbers.

Given that a:b = 31:23

Sum of a and b = a + b = 216

Let the first number ‘a’ = 31x

Then the second number ‘b’ = 23x

Consider a + b = 216

⇒ 31x + 23x = 216

⇒ 54x = 216

⇒ x = 216/54 = 4

∴ The first number ‘a’ = 31x = 31 × 4 = 124

The second number ‘b’ = 23x = 23 × 4 = 92

Q. 4 E. If the product of two numbers is 360 and their ratio is 10 : 9, then find the numbers.

Answer : Let a and b be two numbers.

Given that a:b = 10:9

Product of a and b = ab = 360

Let the first number ‘a’ = 10x

Then the second number ‘b’ = 9x

Consider ab = 360

⇒ 10x × 9x = 360

⇒ $90x^2 = 360$

⇒ $x^2 = 4$

⇒ $x = \pm 2$

If x = 2, then first number ‘a’ = 10x = 10 × 2 = 20

The second number ‘b’ = 9x = 9 × 2 = 18

If x = -2, then first number ‘a’ = 10x = 10 × (-2) = -20

The second number ‘b’ = 9x = 9 × (-2) = -18

Q. 5. If a:b = 3:1 and b:c = 5:1, then find the value of

i. $\left(\frac{a^3}{15b^2c}\right)^3$ ii. $\frac{a^2}{7bc}$

Answer : (i) Given that a:b = 3:1

∴ a/b = 3

⇒ a = 3b ………………… (1)

Also, b:c = 5:1

∴ b/c = 5

⇒ c = b/5 ………………… (2)

Consider $\left(\frac{a^3}{15b^2c}\right)^3$

$$=\left(\frac{(3b)^3}{15b^2\left(\frac{b}{5}\right)}\right)^3$$

$$=\left(\frac{27b^3}{3b^3}\right)^3$$

$=(9)^3$

=729

(ii) Given that a:b = 3:1

∴ a/b = 3

⇒ a = 3b ………………… (1)

Also, b:c = 5:1

∴ b/c = 5

⇒ c = b/5 ………………… (2)

Consider $\frac{a^2}{7bc}$

$$= \frac{(3b)^2}{7b\left(\frac{b}{5}\right)}$$

$$= \frac{9b^2}{\frac{7b^2}{5}}$$

$$= \frac{9b^2 \times 5}{7b^2}$$

$$= \frac{45}{7}$$

Q. 6.

If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$, then find the ratio a/b .

Answer :

Given: $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$

Squaring both sides:

0.04 × 0.4 × a = 0.16 × 0.0016 × b

⇒ 0.016 × a = 0.000256 × b

⇒ a/b = 0.000256/0.016

⇒ a/b = 0.016 = 16/1000

⇒ a/b = 2/125

∴ a:b = 2:125

Q. 7. (x + 3):(x + 11) = (x - 2):(x + 1) then find the value of x.

Answer : Given: (x + 3):(x + 11) = (x - 2):(x + 1)

⇒ (x + 3)/(x + 11) = (x - 2)/(x + 1)

Cross multiply and obtain:

⇒ (x + 3)(x + 1) = (x - 2)(x + 11)

⇒ $x^2 + 4x + 3 = x^2 + 9x - 22$

⇒ 4x + 3 = 9x - 22

⇒ 9x – 4x = 3 + 22

⇒ 5x = 25

⇒ x = 5

∴ x = 5

Practice set 4.3

Q. 1.

If $\frac{a}{b} = \frac{7}{3}$ then find the values of the following ratios.

i. $\frac{5a+3b}{5a-3b}$ ii. $\frac{2a^2+3b^2}{2a^2-3b^2}$

iii. $\frac{a^3+b^3}{b^3}$ iv. $\frac{7a+9b}{7a-9b}$

Answer : Given: a/b = 7/3

∴ a = (7/3)b

(i)

$$\frac{5a+3b}{5a-3b}=\frac{5\times\left(\frac{7}{3}\right)b+3b}{5\times\left(\frac{7}{3}\right)b-3b}$$

$$=\frac{\left(\frac{35b+9b}{3}\right)}{\left(\frac{35b-9b}{3}\right)}$$

$$=\frac{\left(\frac{44b}{3}\right)}{\left(\frac{26b}{3}\right)}$$

$$=\frac{44}{26}$$

$$=\frac{11}{8}$$

(ii)

$$\frac{2a^2+3b^2}{2a^2-3b^2}$$

$$=\frac{2\left(\frac{7}{3}b\right)^2+3b^2}{2\left(\frac{7}{3}b\right)^2-3b^2}$$

$$=\frac{\frac{98b^2}{9}+3b^2}{\frac{98b^2}{9}-3b^2}$$

$$= \frac{\frac{(98b^2 + 27b^2)}{9}}{\frac{(98b^2 - 27b^2)}{9}}$$

$$= \frac{125b^2}{71b^2}$$

$$= \frac{125}{71}$$

(iii)

$$\frac{a^3 - b^3}{b^3}$$

$$= \frac{\left(\frac{7b}{3}\right)^3 - b^3}{b^3}$$

$$= \frac{\frac{343b^3}{27} - b^3}{b^3}$$

$$= \frac{343b^3 - 27b^3}{27b^3}$$

$$= \frac{316b^3}{27b^3}$$

$$= \frac{316}{27}$$

(iv)

$$\frac{7a + 9b}{7a - 9b}$$

$$= \frac{7\left(\frac{7}{3}b\right) + 9b}{7\left(\frac{7}{3}b\right) - 9b}$$

$$= \frac{\frac{49b}{3} + 9b}{\frac{49b}{3} - 9b}$$

$$= \frac{\frac{49b + 27b}{3}}{\frac{49b - 27b}{3}}$$

$$= \frac{76b}{22b}$$

$$= \frac{38}{11}$$

Q. 2.

If $\frac{15a^2 + 4b^2}{15a^2 - 4b^2} = \frac{47}{7}$ then find the values of the following ratios.

i. $\frac{a}{b}$ ii. $\frac{7a - 3b}{7a + 3b}$

iii. $\frac{b^2 + 2b^2}{b^2 - 2b^2}$ iv. $\frac{b^3 + 2b^3}{b^3 - 2b^3}$

Answer :(i) Given:

$$\frac{15a^2+4b^2}{15a^2-4b^2} = \frac{47}{7}$$

Apply componendo and dividendo, i.e., $\left(\frac{a}{b} = \frac{a+b}{a-b}\right)$:

$$\frac{(15a^2 + 4b^2) + (15a^2 - 4b^2)}{(15a^2 + 4b^2) - (15a^2 - 4b^2)} = \frac{47 + 7}{47 - 7}$$

$$\Rightarrow \frac{30a^2}{8b^2} = \frac{54}{40}$$

$$\Rightarrow \frac{a^2}{b^2} = \frac{54 \times 8}{40 \times 30}$$

$$= \frac{9}{25}$$

Take square root on both sides:

$$\Rightarrow \frac{a}{b} = \frac{3}{5}$$

(ii) Given:

$$\frac{15a^2+4b^2}{15a^2-4b^2} = \frac{47}{7}$$

Apply componendo and dividendo, i.e., $\left(\frac{a}{b} = \frac{a+b}{a-b}\right)$:

$$\frac{(15a^2 + 4b^2) + (15a^2 - 4b^2)}{(15a^2 + 4b^2) - (15a^2 - 4b^2)} = \frac{47 + 7}{47 - 7}$$

$$\Rightarrow \frac{30a^2}{8b^2} = \frac{54}{40}$$

$$\Rightarrow \frac{a^2}{b^2} = \frac{54 \times 8}{40 \times 30} = \frac{9}{25}$$

$$\Rightarrow \frac{a}{b} = \frac{3}{5}$$

$$\Rightarrow \frac{7a}{3b} = \frac{21}{15}$$

Again apply componendo and dividendo, i.e., $\left(\frac{a}{b} = \frac{a+b}{a-b}\right)$:

$$\frac{7a + 3b}{7a - 3b} = \frac{21 + 15}{21 - 15}$$

$$= \frac{36}{6}$$

$= 6$

$$\therefore \frac{7a + 3b}{7a - 3b} = 6$$

(iii) Given:

$$\frac{15a^2+4b^2}{15a^2-4b^2} = \frac{47}{7}$$

Apply componendo and dividendo, i.e., $\left(\frac{a}{b} = \frac{a+b}{a-b}\right)$:

$$\frac{(15a^2 + 4b^2) + (15a^2 - 4b^2)}{(15a^2 + 4b^2) - (15a^2 - 4b^2)} = \frac{47 + 7}{47 - 7}$$

$$\Rightarrow \frac{30a^2}{8b^2} = \frac{54}{40}$$

$$\Rightarrow \frac{a^2}{b^2} = \frac{54 \times 8}{40 \times 30} = \frac{9}{25}$$

$$\Rightarrow \frac{a}{b} = \frac{3}{5}$$

$$\Rightarrow \frac{b}{a} = \frac{5}{3}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{25}{9}$$

$$\Rightarrow \frac{b^2}{2a^2} = \frac{25}{18}$$

Again apply componendo and dividendo, i.e., $\left(\frac{a}{b} = \frac{a+b}{a-b}\right)$:

$$\frac{b^2 + 2a^2}{b^2 - 2a^2} = \frac{25 + 18}{25 - 18} = \frac{43}{7}$$

$$\therefore \frac{b^2 - 2a^2}{b^2 + 2a^2} = \frac{7}{43}$$

(iv) Given:

$$\frac{15a^2+4b^2}{15a^2-4b^2}=\frac{47}{7}$$

Apply componendo and dividendo, i.e., $\left(\frac{a}{b}=\frac{a+b}{a-b}\right)$:

$$\frac{(15a^2+4b^2)+(15a^2-4b^2)}{(15a^2+4b^2)-(15a^2-4b^2)}=\frac{47+7}{47-7}$$

$$\Rightarrow \frac{30a^2}{8b^2}=\frac{54}{40}$$

$$\Rightarrow \frac{a^2}{b^2}=\frac{54\times 8}{40\times 30}=\frac{9}{25}$$

$$\Rightarrow \frac{a}{b}=\frac{3}{5}$$

$$\Rightarrow \frac{b}{a}=\frac{5}{3}$$

$$\Rightarrow \frac{b^3}{a^3}=\frac{125}{27}$$

$$\Rightarrow \frac{b^3}{2a^3}=\frac{125}{54}$$

Again apply componendo and dividendo, i.e., $\left(\frac{a}{b}=\frac{a+b}{a-b}\right)$:

$$\frac{b^3+2a^3}{b^3-2a^3}=\frac{125+54}{125-54}=\frac{179}{71}$$

$$\therefore \frac{b^3-2a^3}{b^3+2a^3}=\frac{71}{179}$$

Q. 3. If: $\frac{3a+7b}{3a-7b}=\frac{4}{3}$ then find the value of the ratio $\frac{2a^2-2b^2}{2a^2+7b^2}$.

Answer : Given:

$$\frac{3a+7b}{3a-7b}=\frac{4}{3}$$

Apply componendo and dividendo, i.e., $\left(\frac{a}{b}=\frac{a+b}{a-b}\right)$:

$$\frac{(3a+7b)+(3a-7b)}{(3a+7b)-(3a-7b)}=\frac{4+3}{4-3}$$

$$\Rightarrow \frac{6a}{14b}=7$$

$$\Rightarrow \frac{a}{b}=7\times\frac{14}{6}$$

$$=\frac{49}{3}$$

∴ a/b = 49/3

⇒ a^2/b^2 =2401/9

⇒ $2a^2/7b^2$ =686/9

Apply componendo and dividendo, i.e., $\left(\frac{a}{b}=\frac{a+b}{a-b}\right)$:

$$\frac{2a^2+7b^2}{2a^2-7b^2}=\frac{686+9}{686-9}$$

$$=\frac{695}{677}$$

$$\therefore \frac{2a^2-7b^2}{2a^2+7b^2}=\frac{677}{695}$$

Q. 4 A. Solve the following equations.

$$\frac{x^2+12x-20}{3x-5}=\frac{x^2+8x+12}{2x+3}$$

Answer : Given:

$$\frac{x^2+12x-20}{3x-5} = \frac{x^2+8x+12}{2x+3}$$

We first put x = 0 in the expression and obtain:

$$\frac{(0)^2+12(0)-20}{3(0)-5} = \frac{(0)^2+8(0)+12}{2(0)+3}$$

⇒ -20/-5 = 12/3

⇒ 4=4 which holds true.

∴ x=0 is a solution.

Now, Consider $\frac{x^2+12x-20}{3x-5} = \frac{x^2+8x+12}{2x+3}$ ……………… (1)

Multiply both sides by :

$\frac{x^2+12x-20}{12x-20} = \frac{x^2+8x+12}{8x+12} = k$ ……………….(2)

Apply dividendo:

$$\Rightarrow \frac{x^2+12x-20-(12x-20)}{12x-20} = \frac{x^2+8x+12-(8x+12)}{8x+12}$$

$$\Rightarrow \frac{x^2}{12x-20} = \frac{x^2}{8x+12}$$

$$\Rightarrow \frac{x^2}{12x-20} = \frac{x^2}{8x+12}$$

$$\Rightarrow \frac{1}{12x-20} = \frac{1}{8x+12}$$

Cross multiply and get:

12x – 20 = 8x + 12

⇒ 12x – 8x = 12+ 20

⇒ 4x = 32

$\Rightarrow$ x = 8

$\therefore$ x=0, 8 are the solutions.

Q. 4 B. Solve the following equations.

$$\frac{10x^2 + 15x + 63}{5x^2 - 25x + 12} = \frac{2x + 3}{x - 5}$$

Answer : Given:

$$\frac{10x^2+15x+63}{5x^2-25x+12} = \frac{2x+3}{x-5}$$

We first put x = 0 in the expression and obtain:

$$\frac{10(0) + 15(0) + 63}{5(0) - 25(0) + 12} = \frac{2(0) + 3}{(0) - 5}$$

$\Rightarrow$ 63/12 = 3/(-5) which does not hold true.

$\therefore$ x=0 is not a solution.

Now, let $\frac{10x^2+15x+63}{5x^2-25x+12} = \frac{2x+3}{x-5} = k$ .... (1)

Multiply numerator and denominator of second expression by 5x:

$$\frac{10x^2 + 15x + 63}{5x^2 - 25x + 12} = \frac{10x^2 + 15x}{5x^2 - 25x} = k$$

$$\Rightarrow \frac{10x^2 + 15x + 63 - (10x^2 + 15x)}{5x^2 - 25x + 12 - (5x^2 - 25x)} = k$$

$$\Rightarrow \frac{63}{12} = k$$

$\Rightarrow k = \frac{21}{4}$ ....(2)

$\therefore$ From equation (1) and (2), we get:

$$\frac{2x + 3}{x - 5} = \frac{21}{4}$$

Cross multiply and obtain:

8x + 12 = 21x – 105

⇒ 21x – 8x = 12 + 105

⇒13x = 107

⇒ x = 9

∴ x=2 is the solution.

Q. 4 C. Solve the following equations.

$$\frac{(2x+1)^2+(2x-1)^2}{(2x+1)^2-(2x-1)^2}=\frac{17}{8}$$

Answer : Given:

$$\frac{(2x+1)^2+(2x-1)^2}{(2x+1)^2-(2x-1)^2}=\frac{17}{8}$$

We first put x = 0 in the expression and obtain:

$$\frac{(2(0)+1)^2+(2(0)-1)^2}{(2(0)+1)^2-(2(0)-1)^2}=\frac{17}{8}$$

$$\frac{1+1}{1-1}=\frac{17}{8}$$

⇒ 2/0 = 17/8 which does not hold true.

∴ x=0 is not a solution.

Now, Consider

$$\frac{(2x+1)^2+(2x-1)^2}{(2x+1)^2-(2x-1)^2}=\frac{17}{8}$$

Apply componendo and dividendo:

$$\frac{(2x+1)^2+(2x-1)^2+[(2x+1)^2-(2x-1)^2]}{(2x+1)^2+(2x-1)^2-[(2x+1)^2-(2x-1)^2]}=\frac{17+8}{17-8}$$

$$\Rightarrow \frac{2(2x+1)^2}{2(2x-1)^2} = \frac{25}{9}$$

$$\Rightarrow \left(\frac{2x+1}{2x-1}\right)^2 = \frac{25}{9}$$

Take square roots on both sides:

$$\frac{2x+1}{2x-1} = \pm\frac{5}{3}$$

Case 1:

$$\frac{2x+1}{2x-1} = +\frac{5}{3}$$

Cross multiply and obtain:

6x + 3 = 10x – 5

⇒ 4x = 8

⇒ x = 2

Case 2:

$$\frac{2x+1}{2x-1} = -\frac{5}{3}$$

Cross multiply and obtain:

6x + 3 = -10x + 5

⇒ 16x = 2

⇒ x = 1/8

∴ x = 2, 1/8 are the solutions.

Q. 4 D. Solve the following equations.

$$\frac{\sqrt{4x+1}+\sqrt{x+3}}{\sqrt{4x+1}-\sqrt{x+3}} = \frac{4}{1}$$

Answer : Given:

$$\frac{\sqrt{4x+1}+\sqrt{x+3}}{\sqrt{4x+1}-\sqrt{x+3}}=\frac{4}{1}$$

We first put x = 0 in the expression and obtain:

$$\frac{\sqrt{4(0)+1}+\sqrt{(0)+3}}{\sqrt{4(0)+1}-\sqrt{(0)+3}}=\frac{4}{1}$$

⇒ (1+√3)/(1-√3)=4/1 which does not hold true.

∴ x=0 is not a solution.

Now, Consider

$$\frac{\sqrt{4x+1}+\sqrt{x+3}}{\sqrt{4x+1}-\sqrt{x+3}}=\frac{4}{1}$$

Apply componendo and dividendo:

$$\frac{\left[\sqrt{4x+1}+\sqrt{x+3}\right]+\left[\sqrt{4x+1}-\sqrt{x+3}\right]}{\left[\sqrt{4x+1}+\sqrt{x+3}\right]-\left[\sqrt{4x+1}-\sqrt{x+3}\right]}=\frac{4+1}{4-1}$$

$$\Rightarrow \frac{2\left[\sqrt{4x+1}\right]}{2\left[\sqrt{x+3}\right]}=\frac{5}{3}$$

$$\Rightarrow \sqrt{\frac{4x+1}{x+3}}=\frac{5}{3}$$

Squaring both sides:

$$\frac{4x+1}{x+3}=\frac{25}{9}$$

Cross multiply and get:

36x + 9 = 25x + 75

⇒ 36x -25x = 75 - 9

⇒ 11x = 66

⇒ x = 6

∴ x = 6 is the solution.

Q. 4 E. Solve the following equations.

$$\frac{(4x+1)^2+(2x+3)^2}{4x^2+12x+9}=\frac{61}{36}$$

Answer : Given:

$$\frac{(4x+1)^2+(2x+3)^2}{4x^2+12x+9}=\frac{61}{36}$$

We first put x = 0 in the expression and obtain:

$$\frac{(4(0)+1)^2+(2(0)+3)^2}{4(0)^2+12(0)+9}=\frac{61}{36}$$

$$\Rightarrow \frac{1+9}{9}=\frac{61}{36}$$

⇒ 10/9 = 61/36 which does not hold true.

∴ x=0 is not a solution.

Consider the given equation:

$$\frac{(4x+1)^2+(2x+3)^2}{4x^2+12x+9}=\frac{61}{36}$$

$$\Rightarrow \frac{(4x+1)^2+(2x+3)^2}{(2x+3)^2}=\frac{61}{36}$$

Apply dividendo:

$$\Rightarrow \frac{(4x+1)^2+(2x+3)^2-(2x+3)^2}{(2x+3)^2}=\frac{61-36}{36}$$

$$\Rightarrow \frac{(4x+1)^2}{(2x+3)^2}=\frac{25}{36}$$

Take square root on both sides:

$$\frac{4x+1}{2x+3} = \pm\frac{5}{6}$$

Case 1:

$$\frac{4x+1}{2x+3} = +\frac{5}{6}$$

Cross multiply and obtain:

24x + 6 = 10x + 15

⇒ 14x = 9

⇒ x = 9/14

Case 2:

$$\frac{4x+1}{2x+3} = -\frac{5}{6}$$

Cross multiply and obtain:

24x + 6 = -10x - 15

⇒ 34x = -21

⇒ x = -21/34

∴ x = 9/14 and x = -21/34 are the solutions.

Q. 4 F. Solve the following equations.

$$\frac{(3x-4)^3-(x+1)^3}{(3x-4)^3+(x+1)^3} = \frac{61}{189}$$

Answer : Given:

$$\frac{(3x-4)^3-(x+1)^3}{(3x-4)^3+(x+1)^3} = \frac{61}{189}$$

Apply componendo and dividendo:

$$\frac{(3x-4)^3-(x+1)^3+[(3x-4)^3+(x+1)^3]}{(3x-4)^3-(x+1)^3-[(3x-4)^3+(x+1)^3]}=\frac{61+189}{61-189}$$

$$\Rightarrow \frac{(3x-4)^3}{-(x+1)^3}=\frac{250}{-128}$$

$$\Rightarrow \left(\frac{3x-4}{x+1}\right)^3=\frac{250}{128}=\frac{125}{64}$$

Take cube roots on both sides:

$$\frac{3x-4}{x+1}=\frac{5}{4}$$

Cross multiply and get:

12x - 16 = 5x + 5

⇒ 12x – 5x = 5 + 16

⇒ 7x = 21

⇒ x = 3

∴ x = 3 is the solution.

Practice set 4.4

Q. 1. Fill in the blanks of the following

i. $\frac{x}{7}=\frac{y}{3}=\frac{3x+5y}{\ldots\ldots}=\frac{7x-9y}{\ldots\ldots}$

ii. $\frac{a}{3}=\frac{b}{4}=\frac{c}{7}=\frac{a-2b+3c}{\ldots\ldots}=\frac{\ldots\ldots}{6-8+14}$

(i) $\frac{x}{7}=\frac{y}{3}=\frac{3x}{21}=\frac{5y}{15}=\frac{7x}{49}=\frac{9y}{27}$

$\therefore \frac{3x+5y}{21+15} = \frac{3x+5y}{36}$

$\therefore \frac{7x-9y}{49-27} = \frac{7x-9y}{22}$

Thus, $\frac{x}{7} = \frac{y}{3} = \frac{3x+5y}{36} = \frac{7x-9y}{22}$

(ii) $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{2b}{8} = \frac{3c}{21} = \frac{2a}{6} = \frac{2c}{14}$

$\therefore \frac{a-2b+3c}{3-8+21} = \frac{a-2b+3c}{16}$

$\therefore \frac{2(a)-2(b)+2(c)}{6-8+14} = \frac{2a-2b+2c}{6-8+14}$

Thus, $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{a-2b+3c}{16} = \frac{2a-2b+2c}{6-8+14}$

Q. 2. 5m-n=3m+4n, then find the values of the following expressions.

i. $\frac{m^2+n^2}{m^2-n^2}$ **ii.** $\frac{3m+4n}{3m-4n}$

Answer : (i) Given: 5m – n = 3m + 4m

⇒ 5m – 3m = 4n + n

⇒ 2m = 5n

⇒ m/n = 5/2

⇒ m^2/n^2 = 25/4

Apply componendo and dividendo:

$\therefore \frac{m^2+n^2}{m^2-n^2} = \frac{25+4}{25-4}$

$= \frac{29}{21}$

= 29:21

(ii) Given: 5m – n = 3m + 4m

⇒ 5m – 3m = 4n + n

⇒ 2m = 5n

⇒ m/n = 5/2

⇒ 3m/4n = 15/8

Apply componendo and dividendo:

$$\therefore \frac{3m + 4n}{3m - 4n} = \frac{15 + 8}{15 - 8}$$

$$= \frac{23}{7}$$

= 23: 7

Q. 3 A. If a(y+z) = b(z+x) = c(x+y) and out of a, b, c no two of them are equal then show that

$$\frac{y - z}{a(b - c)} = \frac{z - x}{b(c - a)} = \frac{x - y}{c(a - b)}$$

Answer : Given: a(y+z) = b(z+x) = c(x+y)

Divide all be ‘abc’:

$$\frac{a(y + z)}{abc} = \frac{b(z + x)}{abc} = \frac{c(x + y)}{abc}$$

Cancel out the common factor:

$$\Rightarrow \frac{(y + z)}{bc} = \frac{(z + x)}{ac} = \frac{(x + y)}{ab}$$

Rearrange the terms:

$$\Rightarrow \frac{(x + y)}{ab} = \frac{(y + z)}{bc} = \frac{(z + x)}{ac}$$

Now, subtract third term from first term, subtract first term from second term and subtract second term from third term and obtain the equivalent:

$$\Rightarrow \frac{(x+y)-(z+x)}{ab-ac} = \frac{(y+z)-(x+y)}{bc-ab} = \frac{(z+x)-(y+z)}{ac-bc}$$

Solve and cancel the opposite terms:

$$\Rightarrow \frac{y-z}{a(b-c)} = \frac{z-x}{b(c-a)} = \frac{x-y}{c(a-b)}$$

Hence, proved.

Q. 3 B. If $\frac{x}{3x-y-z} = \frac{y}{3y-z-x} = \frac{z}{3z-x-y}$ **and x + y + z ≠ 0 then show that the value of each ratio is equal to 1.**

Answer : Given: $\frac{x}{3x-y-z} = \frac{y}{3y-z-x} = \frac{z}{3z-x-y}$

Add all the terms and obtain the equivalent:

$$\frac{x}{3x-y-z} = \frac{y}{3y-z-x} = \frac{z}{3z-x-y} = \frac{x+y+z}{3x-y-z+3y-z-x+3z-x-y}$$

$$= \frac{x+y+z}{3x-2x+3y-2y+3z-2z}$$

$$= \frac{x+y+z}{x+y+z}$$

$$= 1$$

∴ Each ratio is equal to 1.

Q. 3 C. If $\frac{ax+by}{x+y} = \frac{bx+az}{x+z} = \frac{ay+bz}{y+z}$ **and x+y+z ≠ 0, then show that ratio is (a+b)/2.**

Answer : Given:

$\frac{ax+by}{x+y} = \frac{bx+az}{x+z} = \frac{ay+bz}{y+z}$

Add all the terms and obtain the equivalent:

$$\frac{ax+by}{x+y} = \frac{bx+az}{x+z} = \frac{ay+bz}{y+z} = \frac{ax+by+bx+az+ay+bz}{x+y+x+z+y+z}$$

$$= \frac{a(x+y+z)+b(x+y+z)}{2(x+y+z)}$$

$$= \frac{(a+b)(x+y+z)}{2(x+y+z)}$$

$$= \frac{a+b}{2}$$

∴ Each ratio is equal to (a+b)/2.

Q. 3 D.

If $\frac{y+z}{a} = \frac{z+x}{b} = \frac{x+y}{c}$ then show that $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$.

Answer :

Given: $\frac{y+z}{a} = \frac{z+x}{b} = \frac{x+y}{c}$ .... (1)

Each ratio $= \frac{z+x+x+y-y-z}{b+c-a} = \frac{2x}{b+c-a}$ .... (2)

Each ratio $= \frac{x+y+y+z-z-x}{c+a-b} = \frac{2y}{c+a-b}$ .... (3)

Each ratio $= \frac{y+z+z+x-x-y}{a+b-c} = \frac{2z}{a+b-c}$ .... (4)

∴ from equation (2), (3) and (4), we get:

$$\frac{2x}{b+c-a}=\frac{2y}{c+a-b}=\frac{2z}{a+b-c}$$

$$\Rightarrow \frac{x}{b+c-a}=\frac{y}{c+a-b}=\frac{z}{a+b-c}$$

Hence, showed.

Q. 3 F. If

$$\frac{3x-5y}{5z+3y}=\frac{x+5z}{y-5x}=\frac{y-z}{x-z}$$

then show that every ratio = x/y.

Answer :

Given: $\frac{3x-5y}{5z+3y}=\frac{x+5z}{y-5x}=\frac{y-z}{x-z}=k\ (say)$

$\frac{3x-5y}{5z+3y}=\frac{x+5z}{y-5x}=\frac{5y-5z}{5x-5z}$ (Multipliy and divide the last term by 5)

Each ratio $=k=\frac{3x-5y+x+5z+5y-5z}{5z+3y+y-5x+5x-5z}$ (Adding all the terms)

$$=\frac{4x}{4y}$$

$$=\frac{x}{y}$$

∴ Each ratio equals x/y.

Q. 4.

Solve.

i. $\frac{16x^2-20x+9}{8x^2+12x+21}=\frac{4x-5}{2x+3}$

ii. $\frac{5y^2+40y-12}{5y^2+10y-4}=\frac{y+8}{1+2y}$

Answer : (i) Given:

$$\frac{16x^2-20x+9}{8x^2+12x+21}=\frac{4x-5}{2x+3}$$

We first put x = 0 in the expression and obtain:

$$\frac{16(0)-20(0)+9}{8(0)+12(0)+21}=\frac{4(0)-5}{2(0)+3}$$

⇒ 9/21 = -5/3 which does not hold true.

∴ x=0 is not a solution.

Now, let $\frac{16x^2-20x+9}{8x^2+12x+21}=\frac{4x-5}{2x+3}=k$ ……………… (1)

Multiply numerator and denominator of second expression by 4x:

$$\frac{16x^2-20x+9}{8x^2+12x+21}=\frac{16x^2-20x}{8x^2+12x}=k$$

$$\Rightarrow \frac{16x^2-20x+9-(16x^2-20x)}{8x^2+12x+21-(8x^2+12x)}=k$$

$$\Rightarrow \frac{9}{21}=k$$

$\Rightarrow k=\frac{3}{7}$ ……………….(2)

∴ From equation (1) and (2), we get:

$$\frac{4x-5}{2x+3}=\frac{3}{7}$$

Cross multiply and obtain:

⇒ 28x – 35 = 6x + 9

⇒ 28x – 6x = 9 + 35

⇒ 22x = 44

⇒ x = 44/22

⇒ x = 2

∴ x=2 is the solution.

(ii) Given:

$$\frac{5y^2+40y-12}{5y^2+10y-4} = \frac{y+8}{1+2y}$$

We first put y = 0 in the expression and obtain:

$$\frac{5(0)+40(0)-12}{5(0)+10(0)-4} = \frac{(0)+8}{1+2(0)}$$

⇒ (-12)/(-4) = 8/1 which does not hold true.

∴ y=0 is not a solution.

Now, let $\frac{5y^2+40y-12}{5y^2+10y-4} = \frac{y+8}{1+2y} = k$ ……………… (1)

Multiply numerator and denominator of second expression by 5y:

$$\frac{5y^2+40y-12}{5y^2+10y-4} = \frac{5y^2+40y}{5y+10y^2} = k$$

$$\Rightarrow \frac{5y^2+40y-12-(5y^2+40y)}{5y^2+10y-4-(5y+10y^2)} = k$$

$$\Rightarrow \frac{-12}{-4} = k$$

$\Rightarrow k = 3$ ……………….(2)

∴ From equation (1) and (2), we get:

$$\frac{y+8}{1+2y}=3$$

Cross multiply and obtain:

⇒ y + 8 = 3 + 6y

⇒ 6y – y = 8 - 3

⇒ 5y = 5

⇒ y = 1

∴ y=1 is the solution.

Practice set 4.5

Q. 1. Which number should be subtracted from 12, 16 and 21 so that resultant numbers are in continued proportion?

Answer : Let x be the number that should be subtracted from 12, 16, 21 so that the numbers remain in continued proportion.

Numbers a, b, c are said to be continued proportion if $b^2 = ac$.

∴ From the definition of continued proportion, we get:

$$\frac{12-x}{16-x}=\frac{16-x}{21-x}$$

⇒ $(16 - x)^2 = (12 - x)(21 - x)$

⇒ $256 + x^2 – 32x = 252 - 33x + x^2$

⇒-32x + 33x = 252 – 256

⇒ x= -4

∴ -4 should be subtracted from 12, 16, 21 so that the numbers remain in continued proportion.

Q. 2. If (28 - x) is the mean proportional of (23 - x) and (19 - x) then find the value of x.

Answer : A number b is said to be mean proportional of two numbers a and c if

$b^2 = ac$.

∴ From the definition of mean proportion, we get:

$(28 - x)^2 = (23 - x)(19 - x)$

$\Rightarrow 784 + x^2 – 56x = 437 - 42x + x^2$

$\Rightarrow -56x + 42x = 437 – 784$

$\Rightarrow -14x = -347$

$\Rightarrow x = 347/14$

Q. 3. Three numbers are in continued proportion, whose mean proportional is 12 and the sum of the remaining two numbers is 26, then find these numbers.

Answer : Let the numbers be x, y, z.

As the numbers are in continued proportion, therefore

$y^2 = xz$ ……………… (1)

Also, the mean proportion = 12

∴ $y = \sqrt{xz} = 12$

$\Rightarrow xz = 144$ …………… (2)

It is given that the sum of remaining two numbers = 26

∴ $x + z = 26$

$\Rightarrow x = 26 – z$

Put the value of x in equation (2):

$(26 – z)z = 144$

$\Rightarrow 26z – z^2 = 144$

$\Rightarrow z^2 – 26z + 144 = 0$

$\Rightarrow z^2 – 8z – 18z + 144 = 0$

$\Rightarrow z(z - 8) – 18(z – 8) = 0$

$\Rightarrow$ (z - 8)(z - 18) = 0

$\Rightarrow$ z = 8 or z = 18

$\therefore$ x = 26 – 8 or x = 26 – 18

$\Rightarrow$ x = 18 or x = 8

y = 12

$\therefore$ The numbers in proportion be 8, 12, 18 or 18, 12, 8.

Q. 4. If (a + b+ c)(a –b + c) = $a^2 + b^2 + c^2$, show that a, b, c are in continued proportion.

Answer : Given: (a + b+ c)(a –b + c) = $a^2 + b^2 + c^2$

$\Rightarrow a^2 - ab + ac + ab - b^2 + bc + ca - bc + c^2 = a^2 + b^2 + c^2$

$\Rightarrow a^2 - ab + ac + ab - b^2 + bc + ca - bc + c^2 - a^2 - c^2 = b^2 + b^2$

$\Rightarrow 2ac = 2b^2$

$\Rightarrow b^2 = ac$

$\therefore$ a, b, c are in continued proportion.

Q. 5. If $\frac{a}{b} = \frac{b}{c}$ **and a, b, c >0, then show that,**

i. (a + b+ c)(b – c) = ab – c^2
ii. ($a^2 + b^2$)($b^2 + c^2$) = (ab + bc)^2
iii. ($a^2 + b^2$)/ab = (a + c)b

Answer : (i) Given: a/b = b/c

$\Rightarrow b^2 = ac$

Consider (a + b+ c)(b – c) = $ab - ac + b^2 - bc + cb - c^2$

= $ab - ac + ac - c^2$ ($\because b^2 = ac$)

= $ab - c^2$

(ii) Given:

$a/b = b/c$

$\Rightarrow b^2 = ac$

Consider $(a^2 + b^2)(b^2 + c^2) = a^2b^2 + a^2 c^2 + b^2 b^2 + b^2c^2$

$= a^2b^2 + ac(ac) + b^2(ac) + b^2c^2$ ($\because b^2 = ac$)

$= a^2b^2 + b^2(ac) + b^2(ac) + b^2c^2$ ($\because b^2 = ac$)

$= a^2b^2 + 2b^2(ac) + b^2c^2$

$= a^2b^2 + 2ab^2c + b^2c^2$

$= (ab + bc)^2$

(iii) Given: $a/b = b/c$

$\Rightarrow b^2 = ac$

Consider $(a^2 + b^2)/ab = (a^2 + ac)/ab$ ($\because b^2 = ac$)

$= (a + c)/b$

Q. 6. Find mean proportional of

$$\left(\frac{x+y}{x-y}\right), \left(\frac{x^2-y^2}{x^2y^2}\right)$$

Answer : Mean proportion of two numbers is the square root of their product.

$\therefore$ Mean proportion of $\frac{x+y}{x-y}, \frac{x^2-y^2}{x^2y^2}$ is:

$$= \sqrt{\left(\frac{x+y}{x-y}\right) \times \left(\frac{x^2-y^2}{x^2y^2}\right)}$$

$$= \sqrt{\left(\frac{x+y}{x-y}\right) \times \left(\frac{(x+y)(x-y)}{x^2y^2}\right)}$$

$= \sqrt{\left(\frac{x+y}{xy}\right)^2}$

$= \frac{x+y}{xy}$

Problem set 4

Q. 1 A. Select the appropriate alternative answer for the following questions.

If 6 : 5 = y : 20 then what will be the value of y?

A. 15
B. 24
C. 18
D. 22.5

Answer : Given: 6:5 = y:20

⇒ 6/5 = y/20

Cross multiply and get:

5y = 6 × 20

⇒ y = 6 × 4 = 24

∴ Option B is correct.

Q. 1 B. Select the appropriate alternative answer for the following questions.

What is the ratio of 1 mm to 1 cm?

A. 1 : 100

B. 10 : 1
C. 1 : 10
D. 100 : 1

Answer : 1 cm = 100 mm

∴ 1mm : 1cm

⇒ 1mm : 100mm

= 1 : 100

∴ Option A is correct.

Q. 1 C. Select the appropriate alternative answer for the following questions.

The ages of Jatin, Nitin and Mohasin are 16, 24 and 36 years respectively. What is the ratio of Nitin's age to Mohasin's age?

A. 3 : 2
B. 2 : 3
C. 4 : 3
D. 3 : 4

Answer : Given: Nitin's age = 24 years

Mohasin's age = 36 years

∴ Ration of Nitin's age to Mohasin's age = 24:36

= 24/36

= 2/3

= 2:3

∴ Option B is correct.

Q. 1 D. Select the appropriate alternative answer for the following questions.

24 Bananas were distributed between Shubham and Anil in the ratio 3 : 5, then how many bananas did Shubham get?

A. 8
B. 15
C. 12
D. 9

Answer : Total bananas = 24

Ratio in which the bananas are divided = 3:5

Let number of bananas Shubham got = 3x

∴ Number of bananas Anil got = 5x

∴ 3x+5x = 24

⇒ 8x = 24

⇒ x = 3

∴ Shubham got (3 × 3) = 9 bananas.

Thus, option D is correct.

Q. 1 E. Select the appropriate alternative answer for the following questions.

What is the mean proportional of 4 and 25?

A. 6
B. 8
C. 10
D. 12

Answer : Mean proportional of two numbers a and b = √(ab)

∴ Mean proportional of 4 and 25 = √(4 × 25)

= √100 = 10

Q. 2. For the following numbers write the ratio of first number to second number in the reduced form.

i. 21, 48 ii. 36, 90
iii. 65, 117 iv. 138, 161
v. 114, 133

Answer : (i) Ratio of 21 and 48 in the reduced form:

$$\frac{21}{48} = \frac{21 \times 1}{21 \times 4}$$

(To simplify, break the numbers in simpler form)

$$= \frac{1}{4}$$

∴ Ratio of 21 and 48 in reduced form is 1:4.

(ii) Ratio of 36 and 90 in the reduced form:

$$\frac{36}{90} = \frac{18 \times 2}{18 \times 5}$$

(To simplify, break the numbers in simpler form)

$$= \frac{2}{5}$$

∴ Ratio of 36 and 90 in reduced form is 2:5.

(iii) Ratio of 65 and 117 in the reduced form:

$$\frac{65}{117} = \frac{13 \times 5}{13 \times 9}$$

(To simplify, break the numbers in simpler form)

$$= \frac{5}{9}$$

∴ Ratio of 65 and 117 in reduced form is 5:9.

(iv) Ratio of 138 and 161 in the reduced form:

$$\frac{138}{161} = \frac{23 \times 6}{23 \times 7}$$

(To simplify, break the numbers in simpler form)

$$= \frac{6}{7}$$

∴ Ratio of 138 and 161 in reduced form is 6:7.

(v) Ratio of 114 and 133 in the reduced form:

$\frac{114}{133} = \frac{19 \times 6}{19 \times 7}$ (To simplify, break the numbers in simpler form)

$$= \frac{6}{7}$$

∴ Ratio of 114 and 133 in reduced form is 6:7.

Q. 3. Write the following ratios in the reduced form.

i. Radius to the diameter of a circle.
ii. The ratio of diagonal to the length of a rectangle, having length 4 cm and breadth 3 cm.
iii. The ratio of perimeter to area of a square, having side 4 cm.

Answer : (i) Let r be the radius of the circle.

Let d be the diameter of the circle.

Diameter = 2 × Radius

∴ Ratio of radius to diameter in the reduced form = Radius:Diameter

$$\frac{\text{Radius}}{\text{Diameter}} = \frac{r}{2r} = \frac{1}{2}$$

∴ Ratio of radius to diameter in the reduced form = 1:2

(ii) Given: Length of rectangle = l = 4 cm

Breadth of rectangle = b = 3 cm

Diagonal of rectangle = $\sqrt{(l^2 + b^2)}$

= $\sqrt{(16 + 9)}$

= $\sqrt{25}$ = 5

∴ Diagonal of rectangle = 5 cm

Ratio of diagonal to the length of a rectangle = 4:5

(iii) Given: Side of square = 4 cm

Perimeter of square = 4 × Side = 4 × 4 = 16 cm^2

Area of the square = $(\text{Side})^2 = (4)^2$ = 14 cm^2

The ratio of perimeter to area of a square = 16:14 = 8:7

Q. 4. Check whether the following numbers are in continued proportion.

i. 2, 4, 8
ii. 1, 2, 3

iii. 9, 12, 16
iv. 3, 5, 8

Answer : (i) Three numbers ‘a’, ‘b’ and ‘c’ are said to be continued proportion if a, b and c are in proportion,

i.e. a:b::b:c

or $b^2 = ac$

Here, a = 2, b = 4 and c = 8

∴ $(4)^2 = 2 \times 8$

⇒ 16 = 16, which holds true.

∴ 2, 4, 8 are in continued proportion.

(ii) Three numbers ‘a’, ‘b’ and ‘c’ are said to be continued proportion if a, b and c are in proportion,

i.e. a:b::b:c

or $b^2 = ac$

Here, a = 1, b = 2 and c = 3

∴ $(2)^2 = 1 \times 3$

⇒ 4 = 3, which does not hold true.

∴ 1, 2, 3 are not in continued proportion.

(iii) Three numbers ‘a’, ‘b’ and ‘c’ are said to be continued proportion if a, b and c are in proportion,

i.e. a:b::b:c

or $b^2 = ac$

Here, a = 9, b = 12 and c = 16

∴ $(12)^2 = 9 \times 16$

⇒ 144 = 144, which holds true.

∴ 9, 12, 16 are in continued proportion.

(iv) Three numbers 'a', 'b' and 'c' are said to be continued proportion if a, b and c are in proportion,

i.e. a:b::b:c

or $b^2 = ac$

Here, a = 3, b = 5 and c = 8

∴ $(5)^2 = 3 \times 8$

⇒ 25 = 24, which does not hold true.

∴ 3, 5, 8 are not in continued proportion.

Q. 5. a, b ,c are in continued proportion. If a = 3 and c = 27 then find b.

Answer : Given: a, b, c are in continued proportion.

Three numbers 'a', 'b' and 'c' are said to be continued proportion if a, b and c are in proportion,

i.e. a:b::b:c

or $b^2 = ac$

Here, a = 3, c = 27

∴ $(b)^2 = 3 \times 27$

⇒ $b^2 = 81$

⇒ $b = \pm\sqrt{81} = \pm 9$

∴ b = -9 or 9

Q. 6. Convert the following ratios into percentages.

i. 37: 500 **ii. 5/8**
iii. 22/30 **iv. 5/16**
v. 144/1200

Answer : (i) 37: 500 = 37/500

= ((37/500) × 100)%

= (37/5) %

= 7.4 %

(ii) 5/8 = ((5/8) × 100)%

= (5 × 12.5) %

= 62.5 %

(iii) 22/30 = ((22/30) × 100)%

= (220/3) %

= 73.33 %

(iv) 144/1200 = ((144/1200) × 100)%

= (144/12) %

= 12 %

Q. 7. Write the ratio of first quantity to second quantity in the reduced form.

i. 1024 MB, 1.2 GB [(1024 MB = 1 GB)]
ii. 17 Rupees, 25 Rupees 60 paise
iii. 5 dozen, 120 units
iv. 4 sq.m, 800 sq.cm
v. 1.5 kg, 2500 gm

Answer : (i) 1024MB = 1 GB

Reduced form of the ratio of 1 GB and 1.2 GB is:

$$\frac{1}{1.2} = \frac{10}{12}$$

$$= \frac{5}{6}$$

∴ The ratio in reduced form is 5:6.

(ii) 60 paise = 0.60 Rupees

∴ 25 Rupees and 60 paise = 25.60 Rupees

Reduced form of the ratio of 17 Rupees and 25.60 Rupees is:

$$\frac{17}{25.60} = \frac{1700}{2560}$$

$= \frac{20 \times 85}{20 \times 128}$ (Break the numbers in simpler form)

$$= \frac{85}{128}$$

∴ The ratio in reduced form is 85:128.

(iii) 1 dozen = 12 units

∴ 5 dozens = 5 × 12 = 60 units

Reduced form of the ratio of 5 dozens (= 60 units) and 120 units is:

$$\frac{60}{120} = \frac{60 \times 1}{60 \times 2}$$

$$= \frac{1}{2}$$

∴ The ratio in reduced form is 1:2.

(iv) 4 sq m = 4 m^2=4$(100cm)^2$

∴ 4 sq m = 40000 sq cm

Reduced form of the ratio of 4 sq m(=40000 sq cm) and 800 sq cm is:

$\frac{40000}{800} = \frac{800 \times 50}{800 \times 1}$ (Break the numbers in simpler form)

$$= \frac{50}{1}$$

∴ The ratio in reduced form is 50:1.

(v) 1 kg = 1000 gm

∴ 1.5 kg = 1500 gm

Reduced form of the ratio of 1500 gm and 2500 gm is:

$$\frac{1500}{2500} = \frac{15 \times 100}{25 \times 100}$$

$$= \frac{3}{5}$$

∴ The ratio in reduced form is 3:5.

Q.8.

If $\frac{a}{b} = \frac{2}{3}$ then find the values of the following expressions.

i. $\frac{4a + 3b}{3b}$ ii. $\frac{5a^2 + 2b^2}{5a^2 - 2b^2}$

iii. $\frac{a^3 + b^3}{b^3}$ iv. $\frac{7b - 4a}{7b + 4a}$

Answer : Given: a/b = 2/3

∴ a = (2/3)b

(i)

$$\frac{4a + 3b}{3b} = \frac{2 \times \left(\frac{2}{3}\right) b + 3b}{3b}$$

$$= \frac{\left(\frac{4b + 9b}{3}\right)}{3b}$$

$$= \frac{13b}{9b}$$

$$= \frac{13}{9}$$

(ii)

$$\frac{5a^2+2b^2}{5a^2-2b^2}=\frac{5\left(\frac{2}{3}b\right)^2+2b^2}{5\left(\frac{2}{3}b\right)^2-2b^2}$$

$$=\frac{\frac{20b^2}{9}+2b^2}{\frac{20b^2}{9}-2b^2}=\frac{\frac{(20b^2+18b^2)}{9}}{\frac{(20b^2-18b^2)}{9}}$$

$$=\frac{38b^2}{2b^2}$$

$$=19$$

(iii)

$$\frac{a^3+b^3}{b^3}=\frac{\left(\frac{2b}{3}\right)^3+b^3}{b^3}$$

$$=\frac{\frac{8b^3}{27}+b^3}{b^3}$$

$$=\frac{8b^3+27b^3}{27b^3}$$

$$=\frac{35b^3}{27b^3}$$

$$=\frac{35}{27}$$

(iv)

$$\frac{7b-4a}{7b+4a}=\frac{7b-4\left(\frac{2}{3}b\right)}{7b+4\left(\frac{2}{3}b\right)}$$

$$= \frac{7b - \frac{8b}{3}}{7b + \frac{8b}{3}}$$

$$= \frac{\frac{21b - 8b}{3}}{\frac{21b + 8b}{3}}$$

$$= \frac{13b}{29b}$$

$$= \frac{13}{29}$$

Q. 9. If a, b, c, d are in proportion, then prove that

i. $\frac{11a^2 + 9ac}{11b^2 + 9bd} = \frac{a^2 + 3ac}{b^2 + 3bd}$

ii. $\sqrt{\frac{a^2 + 5c^2}{b^2 + 5d^2}} = \frac{a}{b}$

iii. $\frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{c^2 + cd + d^2}{c^2 - cd + d^2}$

Answer : (i) Given: a, b, c, d are in proportion.

a, b, c, d are in proportion a : b : : c : d ad = bc

i.e. Product of extremes = product of means.

∴ ad = bc

If $\frac{11a^2+9ac}{11b^2+9bd} = \frac{a^2+3ac}{b^2+3bd}$ then:

$(11a^2 + 9ac)(b^2 + 3bd) = (a^2 + 3ac)(11b^2 + 9bd)$

$\Rightarrow 11a^2b^2 + 33a^2bd + 9ab^2c + 27abcd = 11a^2b^2 + 9a^2bd + 33ab^2c + 27abcd$

$\Rightarrow 33a^2bd + 9ab^2c = 9a^2bd + 33ab^2c$

$\Rightarrow 24a^2bd = 24ab^2c$

$\Rightarrow a^2bd = ab^2c$

$\Rightarrow$ ad = bc, which holds true as the numbers are in continued proportion.

$\therefore \frac{11a^2+9ac}{11b^2+9bd} = \frac{a^2+3ac}{b^2+3bd}$

(ii) Given: a, b, c, d are in proportion.

a, b, c, d are in proportion a : b : : c : d ad = bc

i.e. Product of extremes = product of means.

$\therefore$ ad = bc

If $\sqrt{\frac{a^2+5c^2}{b^2+5d^2}} = \frac{a}{b}$

$\Rightarrow \frac{a^2+5c^2}{b^2+5d^2} = \frac{a^2}{b^2}$

then:

$(a^2 + 5c^2)(b^2) = (b^2 + 5d^2)(a^2)$

$\Rightarrow a^2b^2 + 5c^2b^2 = b^2a^2 + 5a^2d^2$

$\Rightarrow 5b^2c^2 = 5a^2d^2$

$\Rightarrow b^2c^2 = a^2d^2$

$\Rightarrow$ bc = ad

$\Rightarrow$ ad = bc, which holds true as the numbers are in continued proportion.

$\therefore \sqrt{\frac{a^2+5c^2}{b^2+5d^2}} = \frac{a}{b}$

(iii) Given: a, b, c, d are in proportion.

a, b, c, d are in proportion a : b : : c : d ad = bc i.e. Product of extremes = product of means.

∴ ad = bc

If $\frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{c^2 + cd + d^2}{c^2 - cd + d^2}$ then:

$(a^2 + ab + b^2)(c^2 - cd + d^2) = (a^2 - ab + b^2)(c^2 + cd + d^2)$

$\Rightarrow a^2c^2 - a^2cd + a^2d^2 + abc^2 - abcd + abd^2 + b^2c^2 - b^2cd + b^2d^2 = a^2c^2 + a^2cd + a^2d^2 - abc^2 - abcd - abd^2 + b^2c^2 + b^2cd + b^2d^2$

$\Rightarrow -2a^2cd + 2abc^2 + 2abd^2 - 2b^2cd = 0$

$\Rightarrow 2abc^2 - 2b^2cd = 2a^2cd - 2abd^2$

$\Rightarrow 2bc[ac - bd] = 2ad[ac - bd]$

$\Rightarrow 2bc = 2ad$

⇒ ad = bc, which holds true as the numbers are in continued proportion.

$\therefore \frac{a^2 + ab + b^2}{a^2 - ab + b^2} = \frac{c^2 + cd + d^2}{c^2 - cd + d^2}$

Q. 10. If a, b, c are in continued proportion, then prove that

(i) $\frac{a}{a+2b} = \frac{a-2b}{a-4c}$

(ii) $\frac{b}{b+c} = \frac{a-b}{a-c}$

Answer : (i) Given: a, b, c are in continued proportion.

∴ $b^2 = ac$

If $\frac{a}{a+2b} = \frac{a-2b}{a-4c}$ then:

$a(a - 4c) = (a - 2b)(a + 2b)$

$\Rightarrow a^2 - 4ac = a^2 - 4b^2$

$\Rightarrow -4ac = -4b^2$

⇒ $b^2 = ac$, which holds true as the numbers are in continued proportion.

$$\therefore \frac{a}{a+2b} = \frac{a-2b}{a-4c}$$

(ii) Given: a, b, c are in continued proportion.

∴ $b^2 = ac$

If $\frac{b}{b+c} = \frac{a-b}{a-c}$ then:

b(a - c) = (a – b)(b + c)

⇒ ab – bc = ab + ac – b^2 - bc

⇒ ab – bc - ab - ac + bc = -b^2

⇒ -ac = -b^2

⇒ $b^2 = ac$, which holds true as the numbers are in continued proportion.

$$\therefore \frac{b}{b+c} = \frac{a-b}{a-c}$$

Q. 11. Solve:

$$\frac{12x^2 + 18x + 42}{18x^2 + 12x + 58} = \frac{2x+3}{3x+2}$$

Answer : Given:

$$\frac{12x^2 + 18x + 42}{18x^2 + 12x + 58} = \frac{2x+3}{3x+2}$$

We first put x = 0 in the expression and obtain:

$$\frac{12(0) + 18(0) + 42}{18(0) + 12(0) + 58} = \frac{2(0) + 3}{3(0) + 2}$$

⇒ 42/58 = 3/2 which does not hold true.

∴ x=0 is not a solution.

Now, let $\frac{12x^2+18x+42}{18x^2+12x+58} = \frac{2x+3}{3x+2} = k$ ……………… (1)

Multiply numerator and denominator of second expression by 6x:

$$\frac{12x^2+18x+42}{18x^2+12x+58} = \frac{12x^2+18x}{18x^2+12x} = k$$

$$\Rightarrow \frac{12x^2+18x+42-(12x^2+18x)}{18x^2+12x+58-(18x^2+12x)} = k$$

$$\Rightarrow \frac{42}{58} = k$$

$\Rightarrow k = \frac{21}{29}$ ……………….(2)

∴ From equation (1) and (2), we get:

$$\frac{2x+3}{3x+2} = \frac{21}{29}$$

Cross multiply and get:

58x + 87 = 63x + 42

⇒ 63x - 58x = 87 – 42

⇒ 5x = 45

⇒ x = 9

∴ x=9 is the solution.

Q. 12.

If $\frac{2x-3y}{3z+y} = \frac{z-y}{z-x} = \frac{x+3z}{2y-3x}$, then prove that every ratio = x/y.

Answer :

Given: $\frac{2x-3y}{3z+y}=\frac{z-y}{z-x}=\frac{x+3z}{2y-3x}=k\ (say)$

$\frac{2x-3y}{3z+y}=\frac{3y-3z}{3x-3z}=\frac{x+3z}{2y-3x}$ (Multiply and divide the middle term by -3)

Each ratio =k = $\frac{2x-3y+3y-3z+x+3z}{3z+y+3x-3z+2y-3x}=\frac{3x}{3y}=\frac{x}{y}$ (Adding all the terms)

∴ Each ratio equals x/y.

Q.13.

If $\frac{by+cz}{b^2+c^2}=\frac{cz+ax}{c^2+a^2}=\frac{ax+by}{a^2+b^2}$ **then prove that** $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$ **.**

Answer :

$$\frac{by+cz}{b^2+c^2}=\frac{cz+ax}{c^2+a^2}=\frac{ax+by}{a^2+b^2}=k\ (say)$$

Take one term and subtract other two other from it:

$$\therefore \frac{by+cz-(cz+ax)-(ax+by)}{b^2+c^2-(c^2+a^2)-(a^2+b^2)}=\frac{(cz+ax)-(ax+by)-(by+cz)}{(c^2+a^2)-(a^2+b^2)-(b^2+c^2)}$$
$$=\frac{(ax+by)-(by+cz)-(cz+ax)}{(a^2+b^2)-(b^2+c^2)-(c^2+a^2)}$$

$$\Rightarrow \frac{-2ax}{-2a^2}=\frac{-2by}{-2b^2}=\frac{-2cz}{-2c^2}$$

$$\Rightarrow \frac{x}{a}=\frac{y}{b}=\frac{z}{c}$$

Hence, proved.